1/30/18 "After the game, the king and the pawn go into the same box." -Italian Proverb

HW: "Remainder Theorem" HW #1-5
Test 1 on Thursday 2/15

AIM: What are the Remainder and Factor Theorems?

Warm Up:

***Exercise* #1:** Consider each of the following scenarios where we have $\frac{p(x)}{x-a}$. In each case, simplify the division using polynomial long division and then evaluate $p(a)$.

(a) $\frac{x^2-8x+18}{x-2}$

$$\begin{array}{r|rrr} 2 & 1 & -8 & 18 \\ & & 2 & -12 \\ \hline & 1 & -6 & 6 \end{array}$$

$$\boxed{x-6+\frac{6}{x-2}}$$

$p(x)=x^2-8x+18 \Rightarrow p(2)=$

$p(2)=(2)^2-8(2)+18$

$p(2)=6$

(b) $\dfrac{x^2-2x-25}{x-7}$

$$\begin{array}{r} x+5+\frac{10}{x-7} \\ x-7\,\overline{)\,x^2-2x-25} \\ -(x^2-7x)\quad\downarrow \\ \hline 5x-25 \\ -(5x-35) \\ \hline 10 \end{array}$$

$p(x)=x^2-2x-25 \Rightarrow p(7)=$

$p(7)=7^2-2(7)-25$

$p(7)=\boxed{10}$

(c) $\dfrac{2x^2+11x+11}{x+3}$

$$\begin{array}{r|rrr} -3 & 2 & 11 & 11 \\ & & -6 & -15 \\ \hline & 2 & 5 & -4 \end{array}$$

$2x+5+\frac{-4}{x+3}$

$p(x)=2x^2+11x+11 \Rightarrow p(-3)=$

$p(-3)=2(-3)^2+11(-3)+11$

$\boxed{p(-3)=-4}$

(d) $\dfrac{3x^2+7x-20}{x+4}$

$$\begin{array}{r} 3x-5 \\ x+4\,\overline{)\,3x^2+7x-20} \\ -(3x^2+12x)\quad\downarrow \\ \hline -5x-20 \\ -(-5x-20) \\ \hline 0 \end{array}$$

$p(x)=3x^2+7x-20 \Rightarrow p(-4)=$

$p(-4)=3(-4)^2+7(-4)-20$

$p(-4)=0$

THE REMAINDER THEOREM

When the polynomial $p(x)$ is divided by the binomial $(x-a)$ then the remainder will always be $p(a)$. In other words:

$$\frac{p(x)}{(x-a)} = q(x) + \frac{p(a)}{x-a}$$

The remainder when we divide will be the same as if we put the zero into the numerator polynomial

$\frac{x^2-8x+18}{x-2}$ $x-2=0$ $x=2$ $2^2-8(2)+18=6$ ← remainder if we did the division.

Exercise **#2:** If the ratio $\frac{x^2-11x+22}{x-9}$ was placed in the form $q(x)+\frac{r}{x-9}$, where $q(x)$ is a linear function, then which of the following is the value of r?

(1) −3 (3) −9

(2) 5 (4) 4

$x-9=0$
$x=9$
$9^2-11(9)+22$
$81-99+22$
$-18+22$
4

Exercise **#3:** By definition $(x-a)$ is a factor of $p(x)$ if $\frac{p(x)}{x-a}=q(x)$, where $q(x)$ is another polynomial. What must be true of the remainder, $p(a)$, for $(x-a)$ to be a factor of $p(x)$? Explain.

If there is no remainder then the divisor is a factor.

***Exercise* #4:** Determine if each of the following are factors of the listed polynomials by evaluating the polynomials.

(a) Is $x-3$ a factor of $p(x)=x^2-11x+24$?

$x-3=0$
$x=3$

$P(3)=3^2-11(3)+24$
$=9-33+24$
$P(3)=0$

$\boxed{x-3 \text{ is a factor}}$

(b) Is $x+5$ a factor of $p(x)=2x^2+9x-2$?

$x+5=0$
$x=-5$

$P(-5)=2(-5)^2+9(-5)-2$
$=50-45-2$
$P(-5)=3$

$\boxed{x+5 \text{ is NOT a factor}}$

(d) Is $x+1$ a factor of $p(x)=x^3-7x^2-11x-3$

$x=-1$

$P(-1)=(-1)^3-7(-1)^2-11(-1)-3$
$=-1-7+11-3$
$P(-1)=0$

$\boxed{\text{Yes}}$

(c) Is $x-5$ a factor of $p(x)=x^3-x^2-19x-10$?

$x=5$

$P(5)=5^3-5^2-19(5)-10$
$=125-25-95-10$
$P(5)=-5$

$\boxed{\text{NO}}$

7) $\dfrac{x^3+7x^2+13x+3}{x+3}$ has a zero of $x = \underline{-3}$

$$\begin{array}{r|rrrr} -3 & 1 & 7 & 13 & 3 \\ & \downarrow & -3 & -12 & -3 \\ \hline & 1 & 4 & 1 & 0 \end{array}$$

x^2+4x+1

$x^{3}+7x^2+13x+3 = (x+3)(x^2+4x+1)$

$x=-3$

$x = \dfrac{-4 \pm \sqrt{4^2-4(1)(1)}}{2(1)}$

$x = \dfrac{-4 \pm \sqrt{12}}{2}$

$\boxed{x = -3, \dfrac{-4+\sqrt{12}}{2}, \dfrac{-4-\sqrt{12}}{2}}$